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Extending ATP to High Frequencies

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Extending ATP to high frequencies – progress report

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1 Introduction

The Electromagnetic Transients Program (EMTP) and Alternative Transients Program (ATP) contain several modules which calculate the impedance parameters (R', G', L', and C', where the prime indicates per unit length values) of overhead or buried power transmission cables. The calculations in EMTP and ATP are almost entirely based on Carson's 1926 analysis[3] of ground effects on TEM-like modes propagating at low frequencies. In particular, Carson's analysis neglects displacement currents in the ground and radiation from the power lines, both of which can become important at the high frequencies characteristic of broadband over power line (BBPL) applications.

ATP and EMTP will be referred to below jointly as A/EM-TP.

2 ATP interface

A useful interface to A/EM-TP would provide frequency-dependent complex values for the distributed impedance $Z' = R' + i\omega L'$ and distributed admittance $Y' = G' + i\omega C'$ for a single line above a ground plane of relative dielectric constant $\epsilon_2 = \epsilon_r - i\sigma/\omega\epsilon_0$, and for line pairs above that ground plane. Here R', G', L', and $C'$are the resistance in Ω/m, conductance in S/m, inductance in H/m, and capacitance in F/m, respectively, and σ is the conductivity of the ground.

A/EM-TP uses these distributed impedance and admittance values to construct impedance matrices, from which current distributions on the coupled power lines are calculated. For a single wire above an imperfectly conducting ground, the voltage and current are assumed to be described by

$$\frac{dV}{dx} = -Z'I$$

$$\frac{dI}{dx} = -Y'V$$

so that

$$\frac{d^2 2V}{dx^2} = Y'Z'V$$

The complex propagation constant Γ is then given by

$$\Gamma^2 = Y'Z'$$

and the characteristic impedance of the transmission line is

$$Z_0 = \sqrt{\frac{Z'}{Y'}} \, .$$

A/EM-TP also considers pairs of wires above ground, for which the impedance and admittance equations generalize to

$$\frac{dV_1}{dx} = -Z'_{11} I_1 - Z'_{12} I_2 \, ,$$

$$\frac{dV_2}{dx} = -Z'_{21} I_1 - Z'_{22} I_2 \, ,$$

$$\frac{dI_1}{dx} = -Y'_{11} V_1 - Y'_{12} V_2 \, ,$$

$$\frac{dI_2}{dx} = -Y'_{21} V_1 - Y'_{22} V_2 \, .$$

From the single wire and wire pair impedances, A/EM-TP constructs the distributed impedance matrix for multiple overhead power transmission lines, based on the distances between the wires and their height above the ground. The code does not attempt to include effects of bends, jogs, or topography (hills or valleys) – only wire-to-wire distances and wire-to-ground distances, so an extension to higher frequencies is limited in how exact it can or needs to be.

3 Geometry

The papers discussed in what follows use several different Cartesian coordinate systems. We will here translate their discussions to the same coordinate system in which the z-axis is vertical, the ground is at $z \leq 0$ in the $x - y$ plane, and individual wires run parallel to the x-axis at $z = h_k$ (for the k^{th} wire).

4 Carson's transmission line

Carson neglects E_y, E_z, and H_x in the ground, and assumes that the displacement current $\epsilon_2 \frac{\partial E_x}{\partial t}$ is much less then the conductive current σE_x in the ground.

5 Sommerfeld's Hertz dipole

Sommerfeld[1] provides an exact solution for the fields driven by a Hertz dipole above an imperfectly conducting earth. The Hertz dipole is an infinitesimal piece of constant current, which cannot exist in reality because the current does not vanish at the ends but is nonetheless useful for analyzing long lines above the earth. Sommerfeld derives an integral expression that adds to the fields from the dipole at $x = 0, z = h$ and its image at $x = 0, z = -h$ to correct for ground conductivity effects. Sommerfeld's analysis provides the basis for most finite element EM codes, such as NEC and EIGER. Its utility for A/EM-TP is limited, however, because it requires a numerical integration over the horizontal line of current that itself has to be determined self-consistently, complicated by the numerical difficulties of evaluating the Sommerfeld integrals with rapidly oscillating and slowly decaying integrands.

6 Wait and D'Amore

Wait[4] developed integral expressions for the fields around an infinite horizontal wire above a lossy ground based on an assumed $e^{-\Gamma x}$ (for complex Γ) dependence of the current in the wire and the fields. The requirement that the component of the electric field parallel to the wire (E_x) vanish at the wire radius provides the boundary condition to determine Γ. D'Amore and Sarto[11] extended this procedure to account for a lossy wire, and included examples of numerical solutions of the problem. Both papers include expressions for the distributed impedance and admittance of the wire. In [12], the authors extend the calculations to a multiconductor configuration.

The results presented in [11, 12] would appear to be exactly what is required for the interface to a high-frequency extension of A/EM-TP. In a detailed evaluation of the results, however, we encountered two problems. First, the decrease in attentuation above about 3 MHz seen in Fig. 2 of [11] disagrees with numerical simulations of the (nearly) identical problem using NEC2, NEC4, EIGER, and HFSS. The numerical simulations were constructed with lines long enough that reflections from the ends are negligible (because of attenuation) and with a central exciting voltage. The attenuation coefficients calculated numerically do *not* decrease with frequency. A second problem was realized as we were trying to understand the high frequency discrepancies between [11] and the simulations – there is no radiation away from the wire in any of the papers mentioned in this section. The transverse electric and magnetic fields decay exponentially away from the wire, not as $1/\sqrt{r}$ as would be expected for cylindrical radiation.

We believe the two problems outlined in the previous paragraph stem fro the assumptions of 1) an infinite wire, and 2) an $e^{-\Gamma x}$ dependence of the current. If Γ has a positive real part (as it does in the solution), then the current becomes infinite as $x \to -\infty$.

7 Reflection coefficients

The authors of NEC2 included an overly simplified approximation to Sommerfeld's exact theory, treating the ground plane as a reflecting surface, with reflection coefficient for TM fields

$$\mathcal{R}_m = \frac{\epsilon_2 \cos\theta - \left[\epsilon_2 - \sin^2\theta\right]^{1/2}}{\epsilon_2 \cos\theta + \left[\epsilon_2 - \sin^2\theta\right]^{1/2}} \,, \tag{1}$$

where θ is the angle of incidence with respect to the surface normal defined by a line from the image point to the observation point. Comparison between the reflection coefficient option and the Sommerfeld option in NEC2 or NEC4 indicates that the reflection coefficient approximation is very poor for long single wires above a ground plane. We make our own more specific comparison in a later section.

8 Sarabandi's "exact image theory"

Sarabandi, et al.[15] develop a more complete treatment of fields from a horizontal Hertz dipole based on the "exact image theory" of [16]. Defining $\eta = 1/\sqrt{\epsilon_2}$, $\alpha = k_1/\eta$, and $\beta = \eta k_1$, they use the reflection coefficients

$$\Gamma_h = \frac{\eta k_z - k_1}{\eta k_z + k_1}$$

for horizontal (TE to z) polarization, and

$$\Gamma_v = \frac{k_z - \eta k_1}{k_z + \eta k_1}$$

for vertical (TM to z) polarization.

With k_ρ as the integration variable (from 0 to ∞), k_z is defined by $k_z = \sqrt{k_1^2 - k_\rho^2}$. k_z is either real or imaginary, depending on the sign of $k_1^2 - k_\rho^2$. If k_z is taken to be $k_1 \cos\theta$, these reflection coefficients are not quite the same as the usual Fresnel reflection coefficients, as for example in Eq. (1) above, but become quite close for $|\epsilon_2|$ greater than ~4.

The x-component of the direct (from the source) electric field is

$$E_x^i = \frac{-iZ_0 I}{4\pi k_1}\left(k_1^2 + \frac{\partial^2}{\partial x^2}\right)\frac{e^{ik_1 R_0}}{R_0}$$

where $R_0 = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2}$.

The x-component of the image electric field is

$$E_x^{is} = \frac{-iZ_0 I}{4\pi k_1}\left(k_1^2 + \frac{\partial^2}{\partial x^2}\right)\frac{e^{ik_1 R}}{R}$$

where $R = [(x-x')^2 + (y-y')^2 + (z+z')^2]^{1/2}$. The x-component of the diffracted electric field (from ground effects) is then found to be, after some extensive derivation

$$E_x^d = E_x^{is} + \frac{2ik_1 Z_0 I}{4\pi}\left[\frac{e^{ik_1 R}}{R} - \alpha\int_0^\infty e^{-\alpha\xi}\frac{e^{ik_1 R'}}{R'}\,d\xi\right] \tag{2}$$

$$+ \frac{2iZ_0 I\eta}{4\pi(1-\eta^2)}\frac{\partial^2}{\partial x^2}\int_0^\infty \left(e^{-\alpha\xi} - \eta^2 e^{-\beta\xi}\right)\frac{e^{ik_1 R'}}{R'}\,d\xi \tag{3}$$

with $R' = [(x-x')^2 + (y-y')^2 + (z+z'-i\xi)^2]^{1/2}$.

The integrands in the integrals over ξ of Eq. (3) are slowly varying and rapidly convergent, so can be evaluated much more efficiently than Sommerfeld's integrals. These fields provide a good approximation to the exact fields from Sommerfeld's formulation, but permit rapid and computationally efficient integration.

9 Finite-element modeling

Poljak[17] provides a clear description of how to extend the fields from a horizontal Hertz dipole to a finite-element model of a thin wire above a lossy ground. The technique involves solving for unknown currents in wire segments (the finite elements) that give an E_x that vanishes along the wire *except* at the single point of excitation. The combination of this finite element modeling with the E_x expressions of [15] provides the possibility of numerical evaluation of the propagation constant and distributed impedance and admittance for single wires above ground and wire pairs above ground – precisely what A/EM-TP requires for modeling power lines.

We have implemented this numerical procedure and are in the process of evaluating its utility for determining power line impedances and admittances at high frequencies.

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